Understanding Mixed Mode S-Parameters

What the $#%* is a differential or common mode S-Parameter?

The answer to this question is 'A linear Transform'. Essentially, mixed mode S-parameters have been transformed with a simple matrix transformation. The best way to describe this is through an example. Assume a device under test (DUT) is a four port shown in Figure 1.

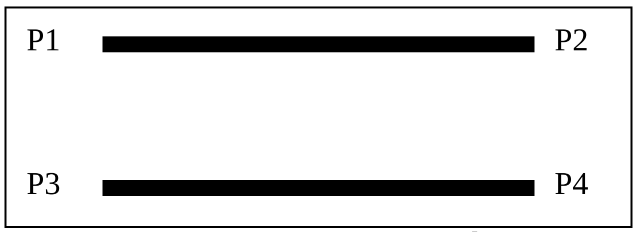

Figure 1: Four Port Device Under test

The device shown in figure 1 can be thought of as two coupled lines. When this device is measured with a Vector Network Analyzer, the S-parameters are a 4 X 4 matrix as shown in Equation 1.

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \qquad \text{Equation 1}$$

Throughout the rest of this example, the S in the matrix will be implied, and just the subscripts will be shown, so Equation 1 (the natural S-parameter matrix) is represented by Equation 2, just the subscripts.

$$\begin{bmatrix} 11 & 12 & 13 & 14 \\ 21 & 22 & 23 & 24 \\ 31 & 32 & 33 & 34 \\ 41 & 42 & 43 & 44 \end{bmatrix} \qquad \text{Equation 2}$$

A quick review of S-parameters may be in order here. Without deriving them, S-parameters (Scattering Parameters) are fairly easy to comprehend. The S-parameter matrix relates the normalized power waves from any port of the DUT to any other port of the DUT. Essentially, throw some power into one port, see how much bounces back (reflection) and how much ends up at the other ports (transmission). That's it.

In the case of mixed mode S-parameters, the excitation is always either odd mode excitation or even mode excitation. Meaning the power going into the device under test is a linear combination of Equation 3.

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \qquad \text{Equation 3}$$

Therefore, it would be useful to create an S-parameter matrix where both the input and the output are linear combinations of Equation 3. This can be accomplished through the use a simple linear transform.

This linear transform is a standard transform and has the following properties.

$$M * S_n * M^{-1} = S_m \qquad \text{Equation 4}$$

where M = transform matrix
S_n = natural S-parameters
S_m = mixed mode S-parameters

$$M * M^{-1} = 1 \qquad \text{Equation 5}$$

The trick is to choose M. From Figure 1, in makes sense to group ports 1 and 3 as one odd/even pair, and ports 2 and 4 as the other odd/even pair. This grouping is accomplished by choosing M as shown in Equation 6.

$$M = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \qquad \text{Equation 6}$$

Using the transform matrix in equation 6 creates the mixed mode matrix shown in equation 7.

$$S_m = \begin{bmatrix} d_1 d_1 & d_1 d_2 & d_1 c_1 & d_1 c_2 \\ d_2 d_1 & d_2 d_2 & d_2 c_1 & d_2 c_2 \\ c_1 d_1 & c_1 d_2 & c_1 c_1 & c_1 c_2 \\ c_2 d_1 & c_2 d_2 & c_2 c_1 & c_2 c_2 \end{bmatrix} \qquad \text{Equation 7}$$

where d1 = differential combination of ports 1 and 3
d2 = differential combination of ports 2 and 4
c1 = common combination of ports 1 and 3
c2 = common combination of ports 2 and 4

To obtain the entries in Equation 7, substitute Equation 6 and Equation 2 into Equation 4.

$$S_m = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 11 & 12 & 13 & 14 \\ 21 & 22 & 23 & 24 \\ 31 & 32 & 33 & 34 \\ 41 & 42 & 43 & 44 \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \quad \text{Equation 8}$$

Combining the two 1/sqrt(2) coefficients and multiplying the first matrix.

$$S_m = \frac{1}{2}\begin{bmatrix} 11-31 & 12-32 & 13-33 & 14-34 \\ 21-41 & 22-42 & 23-43 & 24-44 \\ 11+31 & 12+32 & 13+33 & 14+34 \\ 21+41 & 22+42 & 23+43 & 24+44 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \quad \text{Equation 9}$$

Doing the final matrix multiplication.

$$S_m = \frac{1}{2}\begin{bmatrix} 11-31-13+33 & 12-32-14+34 & 11-31+13-33 & 12-32+14-34 \\ 21-41-23+43 & 22-42-24+44 & 21-41+23-43 & 22-42+24-44 \\ 11+31-13-33 & 12+32-14-34 & 11+31+13+33 & 12+32+14+34 \\ 21+41-23-43 & 22+42-24-44 & 21+41+23+43 & 22+42+24+44 \end{bmatrix} \quad \text{Equation 10}$$

As any good engineer should do, let's take a closer look at Equation 10. One of the easiest cases to investigate is a perfectly matched system with no coupling. In this case, the differential combination and the common combination should be the same, and the off diagonal should equal zero.

In this case:

11 = 22 = 33 = 44
12 = 21 = 34 = 43
14 = 41 = 32 =23 = 0

Putting these constraints into Equation 10 and substituting 11 and 12 where appropriate yeilds:

$$Sm = \frac{1}{2}\begin{bmatrix} 2(11) & 2(12) & 0 & 0 \\ 2(12) & 2(11) & 0 & 0 \\ 0 & 0 & 2(11) & 2(12) \\ 0 & 0 & 2(12) & 2(11) \end{bmatrix} \quad \text{Equation 11}$$

This gave the expected result for the extreme case, so far then Equation 10 is correct.

I'm most interested in the transmission, what will a receiver see, so let's take a closer look at the differential to differential transmission $d_2d_1 = (21 - 41 - 23 + 43)/2$. Graphically, the four paths are shown in Figure 2.

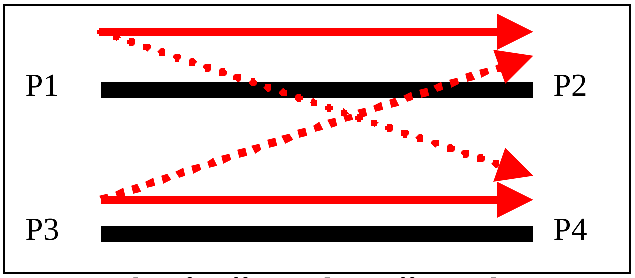

Figure 2: Paths of Differential – Differential Transmission

In Figure 2, the dashed paths are subtracted. Again this makes intuitive sense. The differential transmission is both transmissions added together, minus both the couplings, then the whole thing divided by two. Looking at d_1d_2, it can be seen it is the same as d_2d_1, except the direction of the vectors are opposite. Similarly, the common to common transmission is the same as the differential to differential transmission, except the coupling is added instead of subtracted.

Finally, we'll look at the differential to common transmission, $c_2d_1 = (21 + 41 - 23 - 43)/2$. As before, this can be shown graphically as in Figure 3.

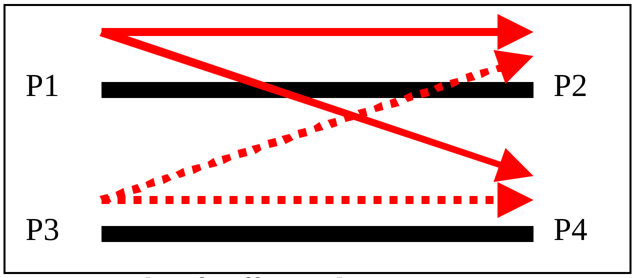

Figure 3: Paths of Differential –Common Transmission

Like Figure 2, the dashed lines in Figure 3 are subtracted. What Figure 3 shows is that if there is either a difference or asymmetry in the two paths, 1 to 2 and 3 to 4, or a difference in the coupling,1 to 4 and 3 to 2, then a common mode component will be generated from a differential input. The magnitude of this common mode component must be investigated to determine what type of common mode rejection is necessary on a differential receiver. Similar to the differential – differential transmission, the cross mode transmission simply changes sign of the coupling terms, or direction of the vectors to obtain c_1d_2, d_1c_2, and d_2c_1.

Let's look at an asymmetric case as shown in Figure 4.

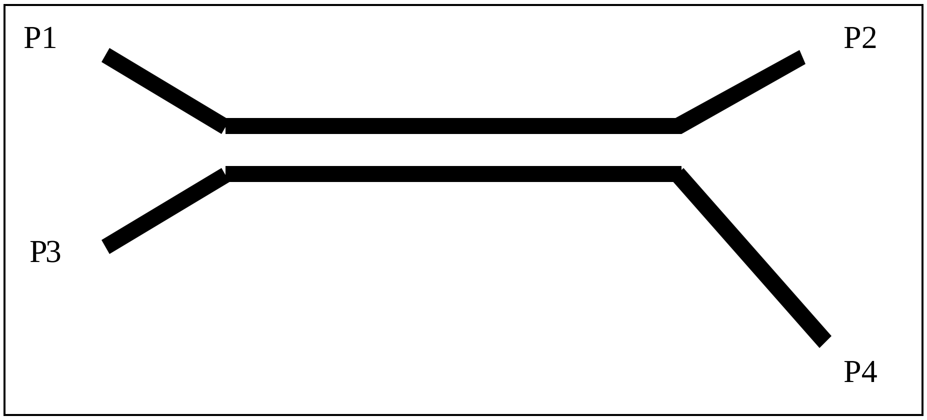

Figure 4: Asymmetric Case

As can be seen in Figure 4, the device under test is a two line system, with one line a little longer than the other going through a coupling area. Therefore, 21 > 43, and 23 > 41. Just to put some arbitrary numbers to these:

21 = 12 = 0.9 V
43 = 34 = 0.8 V
23 = 32 = 0.09 V
41 = 14 = 0.08 V

Therefore:

c_2d_1 = (21 +41 –23 – 43)/2 = (0.9 + 0.08 – 0.09 – 0.8)/2 = 0.045 = -26.9 dB
c_1d_2 = (21 +32 – 14 – 34)/2 = (0.9 + 0.09 – 0.08 – 0.8)/2 = 0.055 = -25.2 dB
d_1c_2 = (12 -32 + 14 – 34)/2 = (0.9 - 0.09 + 0.08 – 0.8)/2 = 0.045 = -26.9 dB
d_2c_1 = (21 - 41 + 23 – 43)/2 = (0.9 - 0.08 + 0.09 – 0.8)/2 = 0.055 = -25.2 dB

What this tells us is that due to the asymmetry in coupling, there can be a larger cross mode coupling in one direction than in the other direction. In this case, there is more differential to common mode conversion from right to the left c_1d_2, than there is from the left to the right c_2d_1. The direction changes for the common to differential conversion, d_2c_1 (left to right) is greater than (d_1c_2) right to left.