

**4.2.2 Surface Microstrip**, Figure 4-3(a) The characteristic impedance,  $Z_{0,surf}$ , of a surface microstrip line is given below[1,2].

$$Z_{0,surf} = \frac{\eta_0}{2\sqrt{2\pi}\sqrt{\epsilon_{r,eff} + 1}} \ln \left\{ 1 + 4 \frac{h}{w'} \left[ 4 \left( \frac{14\epsilon_{r,eff} + 8}{11\epsilon_{r,eff}} \right) \frac{h}{w'} + \sqrt{16 \left( \frac{14\epsilon_{r,eff} + 8}{11\epsilon_{r,eff}} \right)^2 \left( \frac{h}{w'} \right)^2 + \frac{\epsilon_{r,eff} + 1}{2\epsilon_{r,eff}} \pi^2} \right] \right\} \quad (4-4)$$

where  $\eta_0$  is the wave impedance of free space,  $w'$  is the effective signal line width:

$$w' = w + \frac{t}{\pi} \ln \left\{ \frac{4e}{\sqrt{\left( \frac{t}{h} \right)^2 + \left( \frac{t}{w\pi + 1.1t\pi} \right)^2}} \left( \frac{\epsilon_{r,eff} + 1}{2\epsilon_{r,eff}} \right) \right\}, \quad (4-5)$$

and

$$\epsilon_{r,eff} = \begin{cases} \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left\{ \sqrt{\frac{w}{w + 12h}} + 0.04 \left( 1 - \frac{w}{h} \right)^2 \right\} & \frac{w}{h} < 1 \\ \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \sqrt{\frac{w}{w + 12h}} & \frac{w}{h} \geq 1 \end{cases}. \quad (4-6)$$

$W$  is the width of the signal line,  $T$  is the thickness of the signal line,  $H$  is the separation between the signal line and the reference plane, and  $\epsilon_r$  is the relative permittivity of the substrate material. The accuracy of these equations is better than  $\pm 2\%$ [1,2]. For more accuracy, the effect of conductor thickness should be considered[3].

**4.2.3 Embedded Microstrip**, Figure 4-3(b). The effect of embedding the signal line in a single dielectric is to modify (4-4) using a modified  $\epsilon_{r,eff}$  [2]:

$$Z_{0,embedd} = Z_{0,surf} \frac{1}{\sqrt{e^{-2b/h} + \frac{\epsilon_r}{\epsilon_{r,eff}} (1 - e^{-2b/h})}}, \quad (4-7)$$

where  $b$ , see Figure 4-1(d), is:

$$b = h' - h. \quad (4-8)$$

**4.2.4 Symmetric Stripline**, Figure 4-3(c). As was the case for microstrip transmission line, the  $Z_0$  of striplines is also dependent on the ratio of the conductor width and separation between the signal and ground planes. The equations for  $Z_{0,SS}$  of a symmetric stripline are[2]:

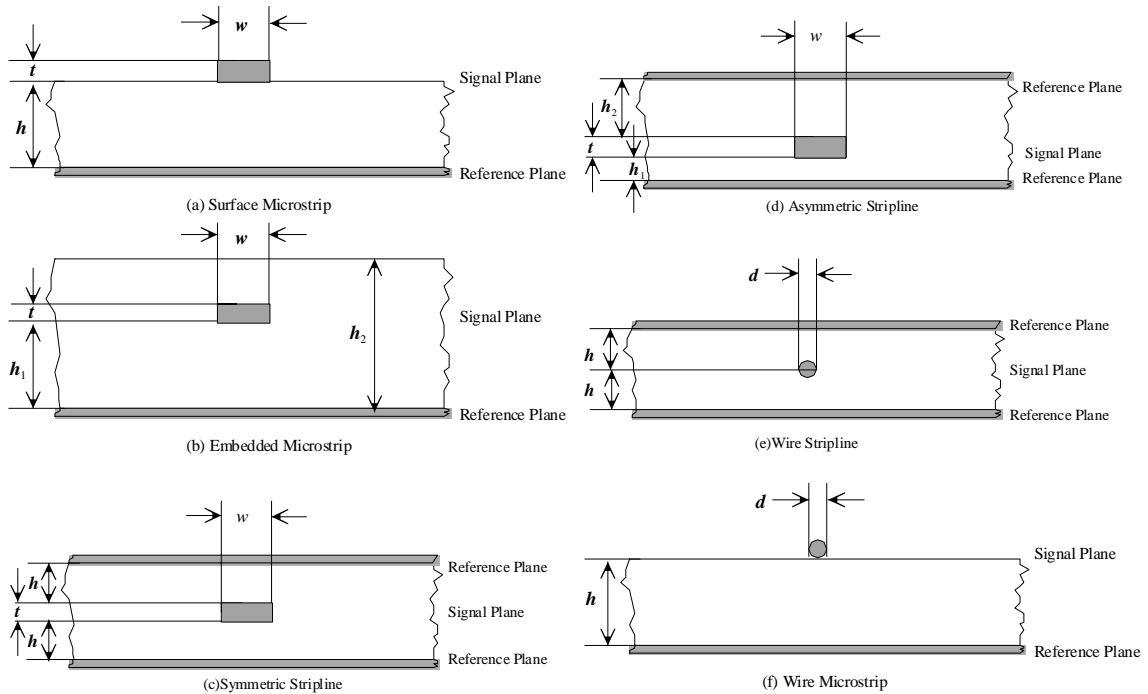
$$Z_{0,SS} = \frac{\eta_0}{2\pi\sqrt{\epsilon_r}} \ln \left\{ 1 + \frac{8h}{\pi w'} \left[ \frac{16h}{\pi w'} + \sqrt{\left(\frac{16h}{\pi w'}\right)^2 + 6.27} \right] \right\}, \quad (4-9)$$

where

$$w' = w + \frac{t}{\pi} \ln \left\{ \frac{e}{\sqrt{\left(\frac{t}{4h+t}\right)^2 + \left(\frac{\pi t}{4(w+1.1t)}\right)^m}} \right\}, \quad (4-10)$$

and

$$m = \frac{6h}{3h+t}. \quad (4-11)$$



**Figure 4-3. Typical unbalanced line configurations**

**4.2.5 Asymmetric Stripline**, Figure 4-3(d). The  $Z_{0,AS}$  for an asymmetric stripline is given by[2]:

$$Z_{0,AS} = \frac{1}{\epsilon_r} \left[ Z_{0,SS}(\epsilon_r = 1, b = h_1 + h_2 + t) - \Delta Z_{0,air} \right], \quad (4-12)$$

where  $Z_{0,SS}(\epsilon_r=1, b=h_1+h_2+t)$  is the  $Z_{0,SS}$  with air as the dielectric and total thickness,  $b$ , equal to  $h_1+h_2+t$ ,  $\Delta Z_{0,air}$  is:

$$\Delta Z_{0,air} = 0.0325\pi Z_{0,air}^2 \left( 0.5 - \frac{1}{2} \frac{2h_1 + t}{h_1 + h_2 + t} \right)^{2.2} \left( \frac{t + w}{h_1 + h_2 + t} \right)^{2.9}, \quad (4-13)$$

where  $h_1$  is the distance between the signal line and the lower reference plane,  $h_2$  is the distance between the signal line and the upper reference plane, and

$$Z_{0,air} = 2 \left[ \frac{Z_{0,SS}(\epsilon_R = 1, b = h_1) Z_{0,SS}(\epsilon_R = 1, b = h_2)}{Z_{0,SS}(\epsilon_R = 1, b = h_1) + Z_{0,SS}(\epsilon_R = 1, b = h_2)} \right], \quad (4-14)$$

where  $Z_{0,SS}(\epsilon_r=1, b=h_1)$  is the  $Z_{0,SS}$  with air as the dielectric and total thickness,  $b$ , equal to  $h_1$ , and  $Z_{0,SS}(\epsilon_r=1, b= h_2)$  is the  $Z_{0,SS}$  with air as the dielectric and total thickness,  $b$ , equal to  $h_2$ .

**4.2.6 Wire Stripline**, Figure 4-3(e). The  $Z_0$  of a transmission line consisting of a circular signal conductor having a diameter  $d$  and centered between parallel ground planes separated by a distance  $h$  is[4]:

$$Z_0 = \frac{\eta_0}{2\pi\sqrt{\epsilon_r}} \ln\left(\frac{4h}{\pi d}\right). \quad (4-15)$$

This equation is accurate to within 1 % for  $d \leq h/2$ [4].

**4.2.7 Wire Microstrip**, Figure 4-3(f).

$$Z_0 = \frac{\eta_0}{2\pi\sqrt{\epsilon_{r,eff}}} \cosh^{-1}\left(\frac{2h}{d}\right) \quad (4-16)$$

where  $d$  is the wire diameter and  $h$  is the separation between the wire and the ground (reference) plane, and  $\epsilon_{r,eff}$  is given by (4-6). To compute  $\epsilon_{r,eff}$  using (4-6),  $d$  is obtained from [2]:

$$\frac{d}{w} = 0.5008 + 1.0235x - 1.023x^2 + 1.1564x^3 - 0.4749x^4, \quad (4-17)$$

where

$$x = \begin{cases} \frac{t}{w} & w \geq t \\ \frac{w}{t} & w < t \end{cases}. \quad (4-18)$$

#### References

1. H.A. Wheeler, "Transmission-Line Properties of a Strip on a Dielectric Sheet on a Plane," IEEE Transactions on Microwave Theory and Techniques, Col. MTT-25, August 1977, pp. 631 - 647.

2. Transmission Line Design Handbook, Brian C. Wadell, Artech House, Norwood, MA 1991.
3. Computer-Aided Design of Microwave Circuits, K.C. Gupta, R. Garg, and R. Chadha, Artech House, Dedham, MA, 1981.
4. S.B. Cohn, "Characteristic Impedance of the Shielded-Strip Transmission Line," IRE Transactions on Microwave Theory and Techniques, July 1954, pp. 52 - 57.
5. M. Kirschning and R.H. Jansen, "Arguments Wide-Range Design Equations for the Frequency-Dependent Characteristic of Parallel Coupled Microstrip Lines," IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-32, January 1984, pp. 83 - 90.
6. S.B. Cohn, "Shielded Coupled-Strip Transmission Line," IRE Transactions on Microwave Theory and Techniques, Vol. MTT-3, October 1955, pp. 29 - 38.
7. P. Bhartia and P. Pramanick, "Computer-Aided Design Models for Broadside-Coupled Striplines and Millimeter-Wave Suspended Substrate Microstrip Lines," IEEE Transactions on Microwave Theory and Techniques, Vol. 36, November 1988, pp. 1476 - 1481.
8. I.J. Bahl and P. Bhartia, "The design of Broadside-Coupled Stripline Circuits," IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-29, February 1981, pp. 165 - 168.